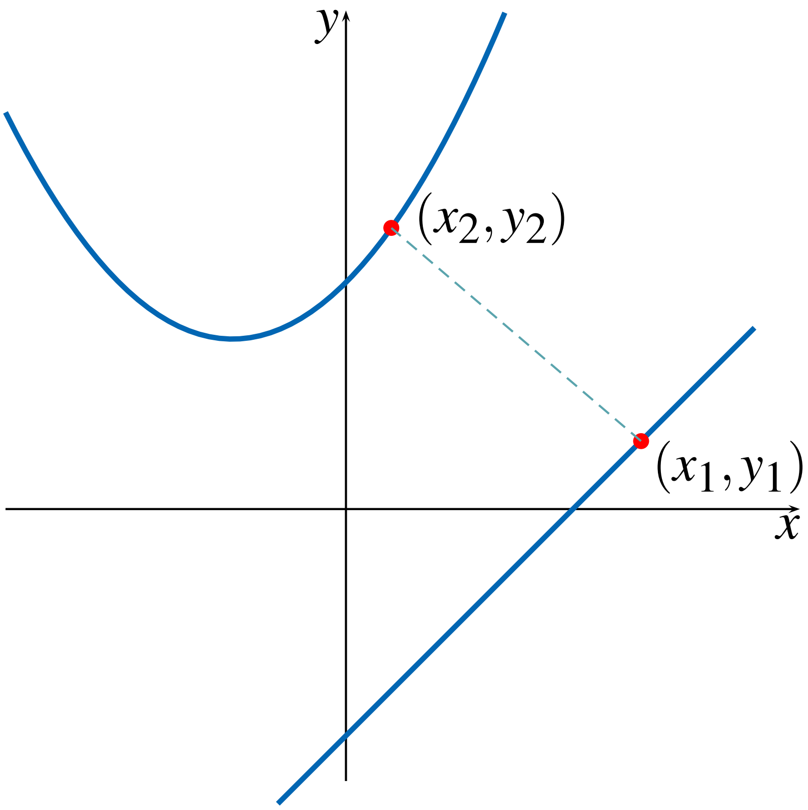
Exemple : Distance entre $y = x^2 + x + 1$ et $y = x - 1$?



$$\min (x_2 - x_1)^2 + (y_2 - y_1)^2$$

$$\begin{cases} g_1 = y_1 - x_1 + 1 = 0 \\ g_2 = y_2 - x_2^2 - x_2 - 1 = 0 \end{cases}$$

- Les fonctions sont C_∞.
- Qualification des contraintes :

$$\mathsf{G} = \begin{pmatrix} \frac{\partial g_1}{\partial x_1} & \frac{\partial g_2}{\partial x_1} \\ \frac{\partial g_1}{\partial y_1} & \frac{\partial g_2}{\partial y_1} \\ \frac{\partial g_1}{\partial x_2} & \frac{\partial g_2}{\partial x_2} \\ \frac{\partial g_1}{\partial y_2} & \frac{\partial g_2}{\partial y_2} \end{pmatrix} = \begin{pmatrix} -1 & 0 \\ 1 & 0 \\ 0 & -2x_2 - 1 \\ 0 & 1 \end{pmatrix}, \qquad \rho(\mathsf{G}) = 2$$

$$L(x_1,y_1,x_2,y_2,\lambda,\mu) = (x_2-x_1)^2+(y_2-y_1)^2 \\ -\lambda(y_1-x_1+1)-\mu(y_2-x_2^2-x_2-1)$$

$$\begin{cases} \dfrac{\partial L}{\partial x_1} = 2(x_1-x_2)+\lambda = 0 \\ \dfrac{\partial L}{\partial y_1} = 2(y_1-y_2)-\lambda = 0, \qquad (x_2-x_1) = (y_1-y_2) \\ \dfrac{\partial L}{\partial x_2} = 2(x_2-x_1)+\mu(2x_2+1) = 0 \\ \dfrac{\partial L}{\partial y_2} = 2(y_2-y_1)-\mu = 0, \qquad (x_2-x_1) = (y_1-y_2)(2x_2+1) \\ \dfrac{\partial L}{\partial \lambda} = -y_1+x_1-1 = 0 \\ \dfrac{\partial L}{\partial \mu} = -y_2+x_2^2+x_2+1 = 0 \end{cases}$$

$$\Rightarrow \qquad x_2 = 0, \quad y_2 = 1, \qquad i.e. \qquad (x_2,y_2) = (0,1)$$

$$\begin{cases} y_1 = 1-x_1 \\ y_1 = x_1-1 \end{cases} \qquad \Rightarrow \qquad (x_1,y_1) = (1,0)$$

$$d = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2} = \sqrt{2}$$

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